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## LIFETIMES OF HIGHLY ECCENTRIC SATELLITES

### INTRODUCTION

King-Hele has provided lifetime estimates (Ref. 1, 2, 3) for the majority of artificial satellite orbits. These are low eccentricity near-earth orbits which contract slowly under the action of air drag until the final phase of rapid decay. The lifetime estimates of satellites orbits primarily disturbed by air drag are calculated from theoretical formulae which determine the rate of change of the orbital period and assume a slow lowering of perigee into the denser regions of the atmosphere (Ref. 4).

Kozai (Ref. 5) found that solar lunar perturbations shortened the lifetime of Explorer VI by a factor of ten. Musen, Bailie and Upton (Ref. 6) found that orbital lifetimes of highly eccentric orbits might be as short as a month, depending on the time of day of launch of an orbit configuration fixed with respect to the earth. For IMP type orbits, where the semimajor axis is a substantial fraction of the moon's, one orbital revolution may be sufficient to lower perigee below the ground if an unfavorable configuration is chosen. Orbits which have lifetimes of at least

one year considering the action of solar and lunar perturbations are preselected for Goddard Satellites with highly eccentric orbits (Ref.8 ) No stipulation on the total lifetime is made. In the present paper, the general characteristics of the lunar solar perturbations will be described and lifetime predictions based on lunar solar perturbations will be made for several Goddard Satellites previously launched.

The major perturbations considered in predicting the lifetime of highly eccentric orbits in the earth moon system are the gravitational asymmetry of the earth and the lunar and solar gravitation. The atmospheric perturbation is not normally computed concomitantly because the assumption is made that perigee height is above the regions of significant drag until the final stage of decay. It is shown that this assumption is not valid for "intermediate" eccentricities.

Numerical integration is used because the elements vary in a complicated manner due to the different forces. Computational results were obtained either from the ITEM program (Ref.9 ), which performs a point-by-point computation of the particle's motion utilizing a modified Encke method of trajectory computation, or from the Halphen method (Ref. 10, 11), which computes the long period perturbation of the moon on elliptic orbits with large eccentricity and inclination.

A major wiggle is easily distinguished in the results of numerical integration having a cycle of  $1/2$  the period of the

motion of the disturbing body. (These wiggles are usually smoothed out in the graphs presented herein). By analogy with general perturbation theory, this is called a "short period term". A biweekly oscillation about the general trend in the orbital elements is induced by the moon, and a semiannual oscillation induced by the sun.

Besides the short period oscillations in perigee height, there is a long period oscillation in perigee height with a period presently unpredicted mathematically. Kozai (Ref.12) has computed the variations of eccentricity and inclination with respect to the argument of perihelion of highly inclined, highly eccentric asteroids. He has presented results showing the wide range amplitudes the eccentricity may take on, the variation becoming more pronounced when  $a/a'$  is non-zero. A functional coupling of the eccentricity and inclination can be obtained by reducing Tisserand's criterion for the identification of comets with the assumption that "a" is a constant. Tisserand's criterion is itself an approximate form of the Jacobi integral of the restricted three-body problem. (Ref.12). This procedure obtains

$$\sqrt{1 - e^2} \cos i = \text{Constant}$$

The formula serves to explain approximately the interesting shapes displayed in Fig. 1, showing the long period variations of  $e$  and  $i$  of a hypothetical earth satellite orbit perturbed solely by the moon. This curve is characteristic of the changes

in  $e$  and  $i$  of a body with high eccentricity or inclination under the long period perturbation due to a body whose orbit encircles the satellite's orbit.

Combining the gravitational perturbation of the sun and the moon produces a change in the period and the amplitude due to the long period perturbation on the eccentricity. This was determined by making three separate computer runs, one with only the sun's gravitational influence, one with only the moon's gravitational influence and one including both effects (Fig. 2).

The earth's oblateness rotates the line of apsides of the near earth satellites. This changes the argument of perigee with respect to the disturbing plane, which Kozai's work indicates determines the phase of the long period perturbation. For "near earth" highly eccentric orbits the changes in  $\Omega$  and  $\omega$  are substantial, about  $0.1^\circ/\text{day}$  for Explorer XVII. Thus the oblateness has an indirect effect on satellite lifetime.

## II. HIGH ECCENTRICITY

EGO, launched September 6, 1964, has a relatively long lifetime among the group considered herein. There is a very strong perigee rise early in the orbital history. (Fig. 3). Apparently the orbit selected happened to be near the minimum of the long period oscillation of perigee height on the rising side. The period of the oscillation is about 16 years. Perigee rises from a height of 285 km to over 50,000 km. The eccentricity has dropped from .92 to .29 at perigee maximum, the inclination at this point is  $60^\circ$  (Fig. 4). The completion of the oscillation brings perigee close to the earth, actually touching earth; the minimum perigee indicated on the printout is 144 km. This is close to the destructive regions of the earth's atmosphere and to the earth's surface. However, there are several sources of uncertainties due to the data. The definition of the initial orbital elements obtained from data depends on the smoothing process. The orbital elements as utilized by the Halphen program are not clearly related to osculating elements (Ref. 11). The Halphen program omits the biweekly moon effect, which has an amplitude on the order of 30 km and a period of two weeks, perigee height is here presented as a smooth curve; plotted from points computed every ten days; actually, perigee height should be represented as a series of dots spaced an orbital period apart (2.7 days). There may be other unknown uncertainties in the prediction. The situation is complicated even more by

the appearance of a second dip toward the earth 160 days later. Here the minimum value of perigee is 173 km. The intermediate maximum value of perigee is 673 km. Apparently, this small fluctuation arises from the semiannual portion of the solar perturbation superposed on the minimum condition of the long period oscillation. All of the uncertainties enumerated above apply to the second value of perigee minimum.

However, if the satellite survives these two minimums, the long period term begins to drive perigee upward again (dotted line). The computation was stopped after 20 years; the portion of the second oscillation computed indicates it to be comparable to the first. In conclusion, the best prediction that can be made in the present study is that the satellite lifetime will be at least 16 years and may be much longer.

The earth's oblateness in this particular case has acted to enhance the lifetime of the satellite. Computations made with the oblateness excluded show a definite termination of the lifetime after 13 years. The oblateness may act to shorten the lifetime of a satellite.

Explorer XIV, like EGO, has a perigee situation which renders the lifetime prediction indeterminate. After 10 years in orbit. the perigee height is lowered to a minimum of 123 km. This is near the critical region of the atmosphere for this satellite (Ref. 7). The computation indicates that the satellite will make several perigee passages during this minimum. If the minimum is higher than this value, the satellite may spend years more in orbit.



The changes in perigee height on the IMP 1, 2, 3 orbits are feeble in comparison with the great changes that occur for some orientations in space. This orbit, whose basic feature is an apogee over  $1/2$  of the lunar distance, is heavily perturbed by the sun and the moon if sufficient time is available. IMP 1, however, has a lifetime of only two years, and the maximum value of perigee height is only 4000. km. The inclination varies between  $33^{\circ}$  and  $40^{\circ}$  with respect to the plane of the earth's equator. IMP 2 failed to achieve the desired initial apogee distance. The resulting orbit, comparable to the Explorer XII orbit, is only mildly perturbed during the two year lifetime. Maximum value of perigee height is 1500. km; inclination variations are within  $3^{\circ}$  degrees. IMP 3 provides a better demonstration of the potential of this orbit. Perigee height attains a value of 35000. km. and the inclination rises to a maximum of  $53^{\circ}$  within the 3 year lifetime. These perturbational changes are still small, however, compared with those occurring on the EGO orbit selected for the first EGO mission.

### III. INTERMEDIATE ECCENTRICITY

The lunar-solar perturbations on the perigee height of two satellite orbits of "intermediate" eccentricity were computed. Explorer XXVI has an initial value of  $e \approx .66$  and a "moderate" semimajor axis = 20,000.km. There is a rapid variation in perigee height of about 70 km in amplitude and one year in period. The interactions between the gravitational perturbations on this orbit are complex. Furthermore, the low level at which perigee is remaining - between 250 and 350 km - and the increased frequency of perigee passages per year indicate that the atmosphere will exert a continuous and non-negligible drag on the orbit. Therefore no estimate of the lifetime will be made herein, and a realistic lifetime prediction must consider the atmosphere force and the gravitational perturbations as acting simultaneously throughout the orbital history. The Explorer XV lifetime prediction (Fig 5 ) would require the same combined calculation. The atmospheric drag, which was omitted in Fig. 5 , is an essential part of the problem.

#### IV. HYPOTHETICAL IMP LIFETIMES

It may be noticed from the lifetime estimate table that the lifetime does not have an obvious relation to the initial orbital elements. That the lifetime is not a well-behaved function of the initial orbital elements is demonstrated by calculating it for a sequence of hypothetical orbits. These orbits have the same initial orbital elements  $a$ ,  $e$ ,  $i$ ,  $\omega$ ;  $i$  and  $\omega$  being defined with respect to the plane of the earth's equator. Only the right ascension of the ascending node is varied, which is equivalent to changing the time of day of launch of an orbit with earth-fixed injection coordinates. The rotation of the earth provides each initial value of node once a day. Two different launch days were chosen. The results, illustrated in Fig. 6, indicate that orbits which existed for over 20 years with a launch date of May 26, 1964, were quickly extinguished if the launch date was February 26, 1964. This is because the short period trend of the solar perturbation had a downward effect on perigee height early in the orbital history of the February 26 launch. Furthermore, the perigee height of these long-lived orbits oscillated between a minimum of 500 - 4000 km and a maximum of 40,000 - 60,000 km with an oscillation period of about 4 years. It can only be supposed that a favorable combination of circumstances held perigee above the earth's atmosphere during these earlier perigee height minimums. Indeed, perigee minimums were occasionally close enough to preclude a definite survival within the limits of the accuracy of the study. Specifically the biweekly lunar term, which was omitted in this calculation, may add further irregularities to the curve in Fig 6

## V. ACCURACY

As indicated already several factors, which are difficult to evaluate, influence the accuracy of the lifetime prediction.

A comparative study of the Halphen and ITEM numerical integration programs has been made by Smith (Ref. 11) for a sample IMP orbit over a three year duration. The results indicate good stability of the Halphen program with some dependency upon the choice of averaged or of osculating elements obtained from ITEM for use as initial conditions. No evaluation of either numerical integration program has been made for a larger interval of time.

The orbital elements used for input into the Halphen program (Table 1) are determined from data over a period of time, that is, they represented smoothed elements rather than initial conditions. The divergence between these will amplify during the orbital history.

The omitted short period moon term may affect the lifetime prediction by a few days either way. When the minimum value of perigee only grazes the atmosphere; instead of descending steadily far below the earth's surface for a prolonged period of time, the short period term is part of the uncertainty in predicting whether or not the satellite will fall.

The uncertainty in the predicted lifetime should depend upon the individual satellite, considering the longevity, the determination of the initial orbital elements, and the manner in which perigee descends into the atmosphere. However, an arbitrary assignment of one month is made to the uncertainty of the predicted lifetimes herein, unless otherwise indicated.

## VI. CONCLUSIONS

Lifetimes of several high eccentricity satellites which are listed with question marks in King-Hele's tables have been obtained with more certainty by including the solar-lunar perturbations. Minimum lifetimes have been found for two satellites, EGO and Explorer XIV. The computations are indeterminate as to whether the lifetime will be equal to or greater than the minimum lifetime. Satellites of "intermediate" eccentricity ( $.4 < e < .7$ ) were shown to be affected by lunar and solar perturbations, but it is likely that they are simultaneously affected by atmospheric drag. These results are summarized in Table I. One satellite, Explorer XII, descended in August 1963; Explorer XVIII will have descended by November 1965.

Musen (Ref. 14) has suggested that artificial satellites will be useful in testing the theories of celestial mechanics. Long period effects on the stability of planetary orbits cannot be observed over full periods because of the length of time necessary to complete a period. In the motion of an artificial satellite the effects may be observed directly and in a relatively short period of time, as if the time scale had been contracted. Of the satellites considered here, EGO I would provide the best test because of the longevity of the orbit and the large amplitude of the changes in the orbital elements. Continued tracking of EGO, if possible, might prove to be of great value to celestial mechanics.

The location and time of the descent of a satellite in a highly elliptical orbit is inherently easier to predict in advance than that of a satellite in a near circular orbit. The atmospheric drag on a satellite in a near earth circular orbits gradually wears the orbit into the denser regions of the atmosphere. Because the process is a small change accumulating over a period of time, the uncertainties in the satellite and drag parameters play an important part in the prediction of descent conditions. Since the orbital decay is a slowly spiraling-in circle, perigee is not well defined and descent may begin along any portion of the orbit. A highly elliptical orbit undergoing perigee decreases of tens or hundreds of kilometers every orbital revolution due to the lunar-solar influence may go from negligible to destructive regions of the atmosphere in one or a few orbits. The location of entry into the atmosphere is confined to a region of the orbit near perigee. The location and the time of descent may, in the future, be predictable enough to station trained observers and equipment in advance of descent. Jacchia's collection of reports by observers who happened to witness the descent of Sputnik 2 provides an interesting account of the breakup and burning of an artificial satellite in the earth's atmosphere (Ref. 15). Perhaps much information of scientific value could be obtained from further observations of objects descending to earth.

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#### FIGURE CAPTIONS

- Figure 1      Eccentricity and inclination vs. time in orbit for a hypothetical IMP orbit disturbed only by the moon.
- Figure 2      Eccentricity vs. time for the hypothetical IMP orbit considering perturbations by the moon only, by the sun only, and by the moon and sun combined.
- Figure 3      Projected OGO-1 Perigee Height.
- Figure 4      Projected inclination of OGO-1 orbit.
- Figure 5      Variations in Explorer XV Perigee Height caused by the moon, sun and oblateness (atmospheric drag not included.)
- Figure 6      Lifetime of a hypothetical IMP orbit with all initial values of the right ascension of ascending node considered.

TABLE I

	Launch Date			Semimajor Axis a (km)	Eccentricity e	Inclination i (deg)	Arg. Perigee $\omega$ (deg)	Rt. AscNode $\Omega$ (deg)	Lifetime	Fall Time	Comments
	Year	Month	Day								
Explorer X	1961	March	21	100,000.	.940	32.9	128.1	-151.8	7 yrs.	April 1968	
Explorer XII	1961	August	16	45,200.	.852	33.0	153.4	171.1	2 yrs.	Aug. 1963	No longer in orbit
Explorer XIV	1962	October	2	55,900.	.881	33.0	150.0	140.6	$\geq 10$ yrs.	Feb. 1973?	May live much longer
Explorer XV	1962	October	27	15,400.	.564	18.0	136.8	-143.8	?	?	Requires drag computation also
Explorer XVIII	1963	November	27	104,000.	.937	33.3	133.1	-101.5	2 yrs.	Oct. 1965	Near termination!
OGO-1 (EGO)	1964	Sept.	9	81,200.	.918	31.1	-46.8	168.5	$\geq 16$ yrs.	1980?	May live much longer
Explorer XXI	1964	October	4	54,000.	.878	33.5	132.9	-135.6	2 yrs.	Sept. 1966	
Explorer XXVI	1964	December	21	20,000.	.659	20.1	121.2	55.0	?	?	Requires drag computation
Explorer XXVIII	1965	May	29	139,000.	.953	33.8	135.7	-138.5	3 yrs.	July 1968	











